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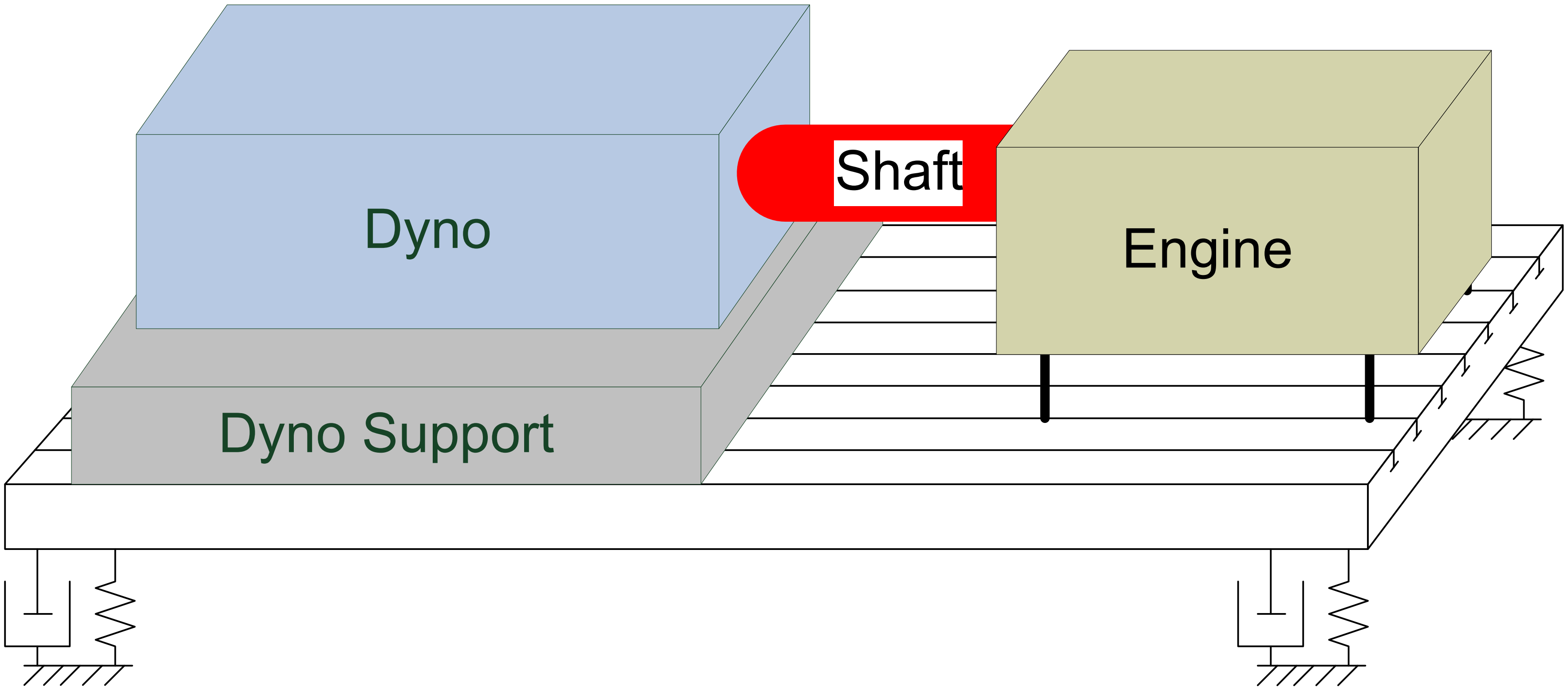
Mechanic Setup



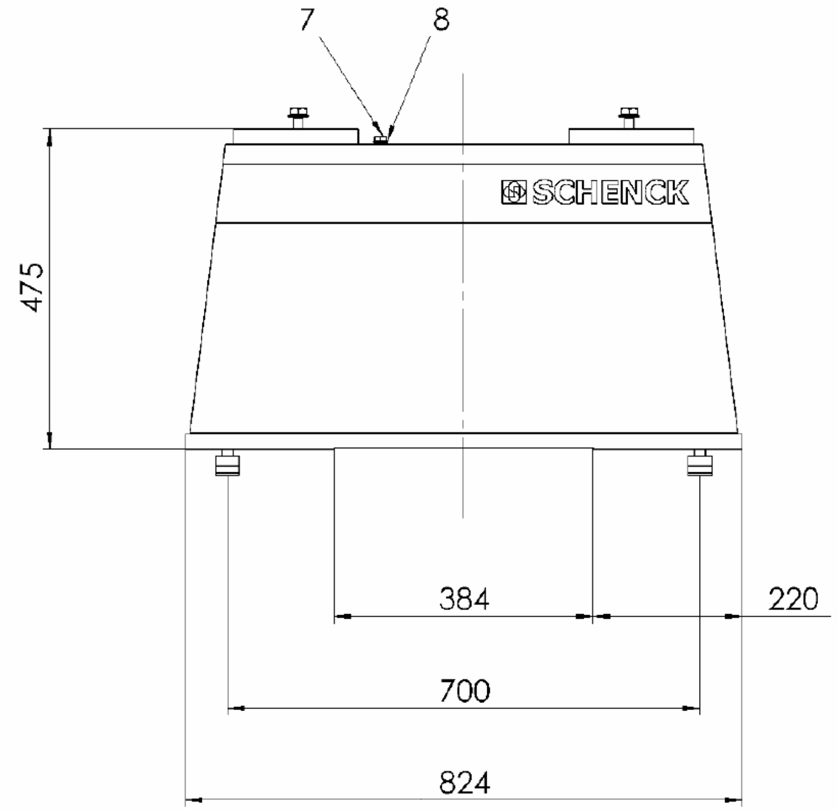
What is Resonance?

4.1 Engine and Dyno Mount, Longitudinal Oscillations

4.2 Shaft, Torsional Oscillations,

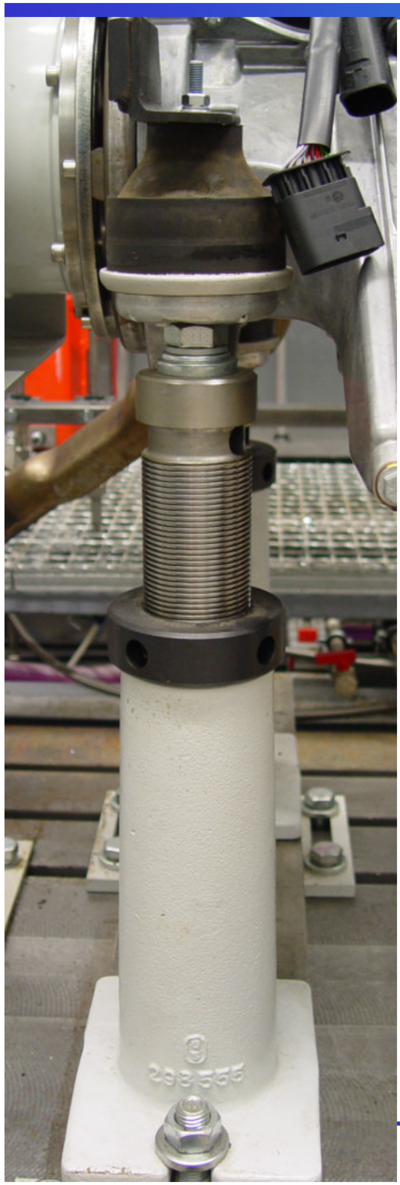
(and Bending Oscillations)

Mechanic Setup
Dyno Support



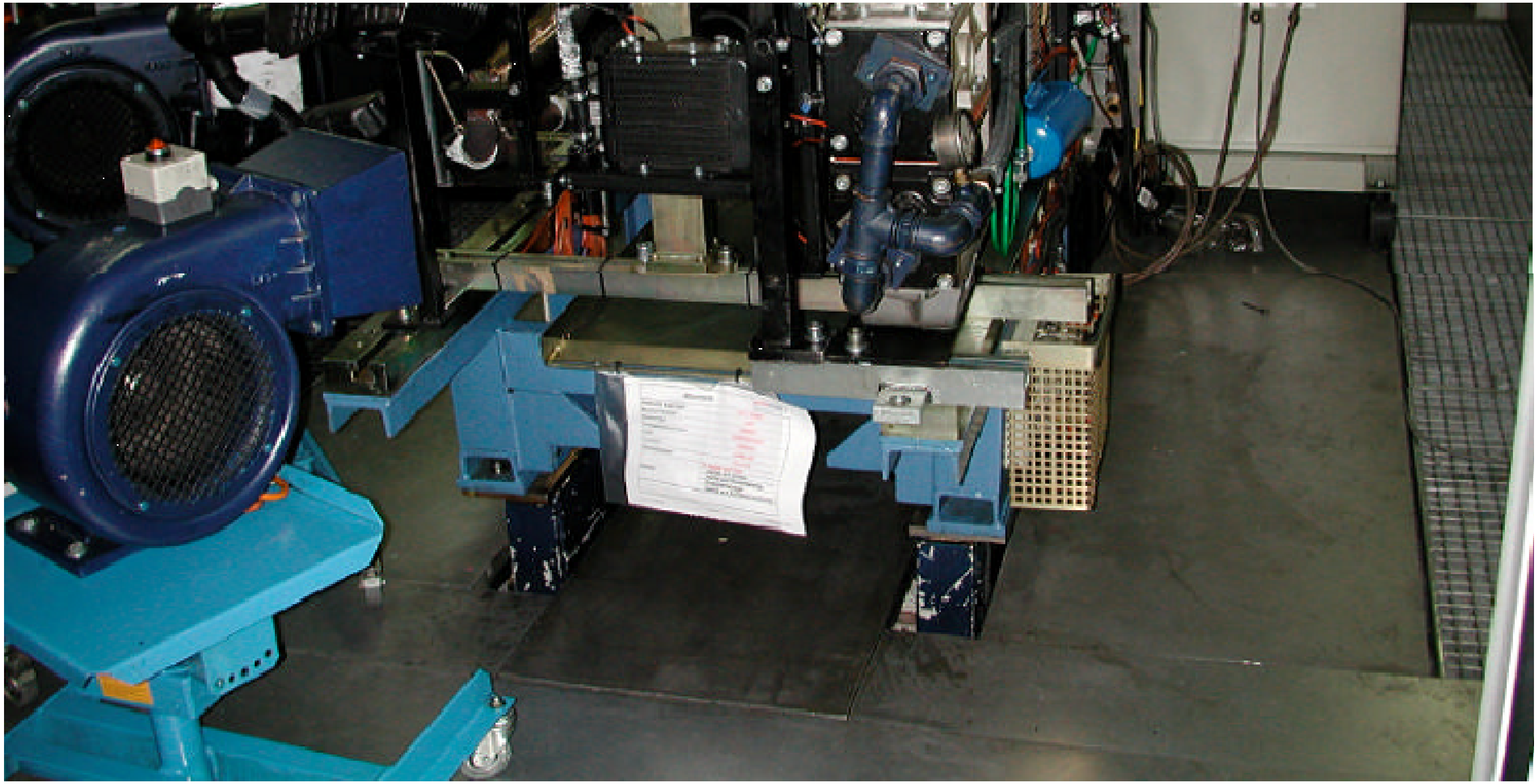
Source:
Horiba

Mechanic Setup
Engine Mount



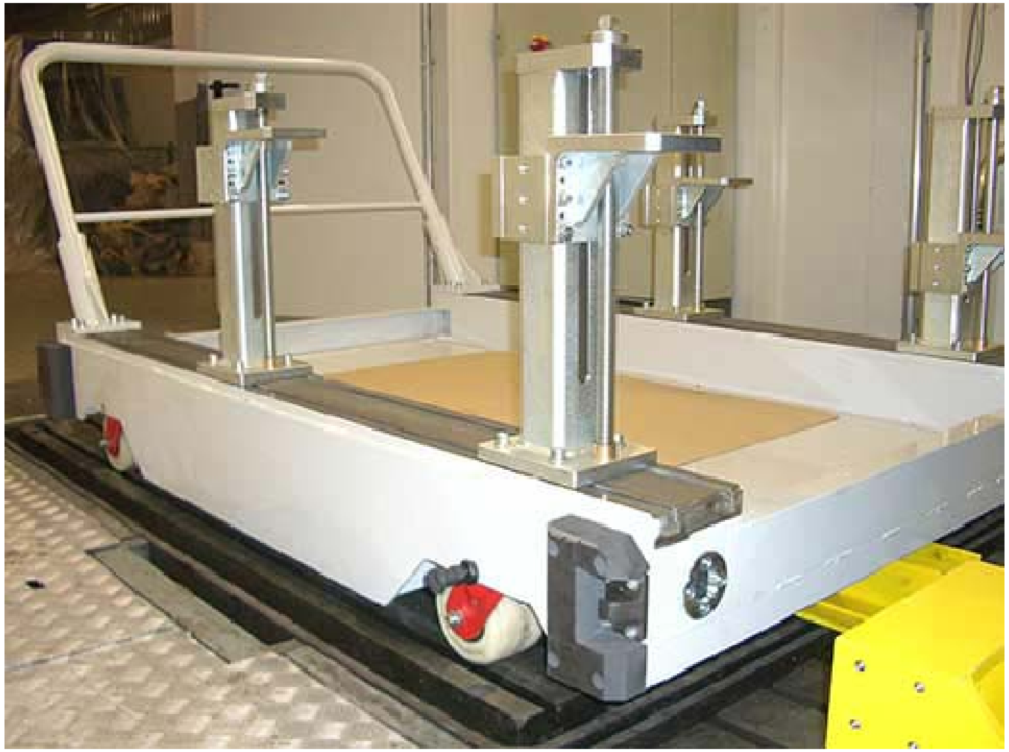
Mechanic Setup
Engine Mount

Palette Mount
(Source: Elring Klinger Motortechnik GmbH, Idstein)



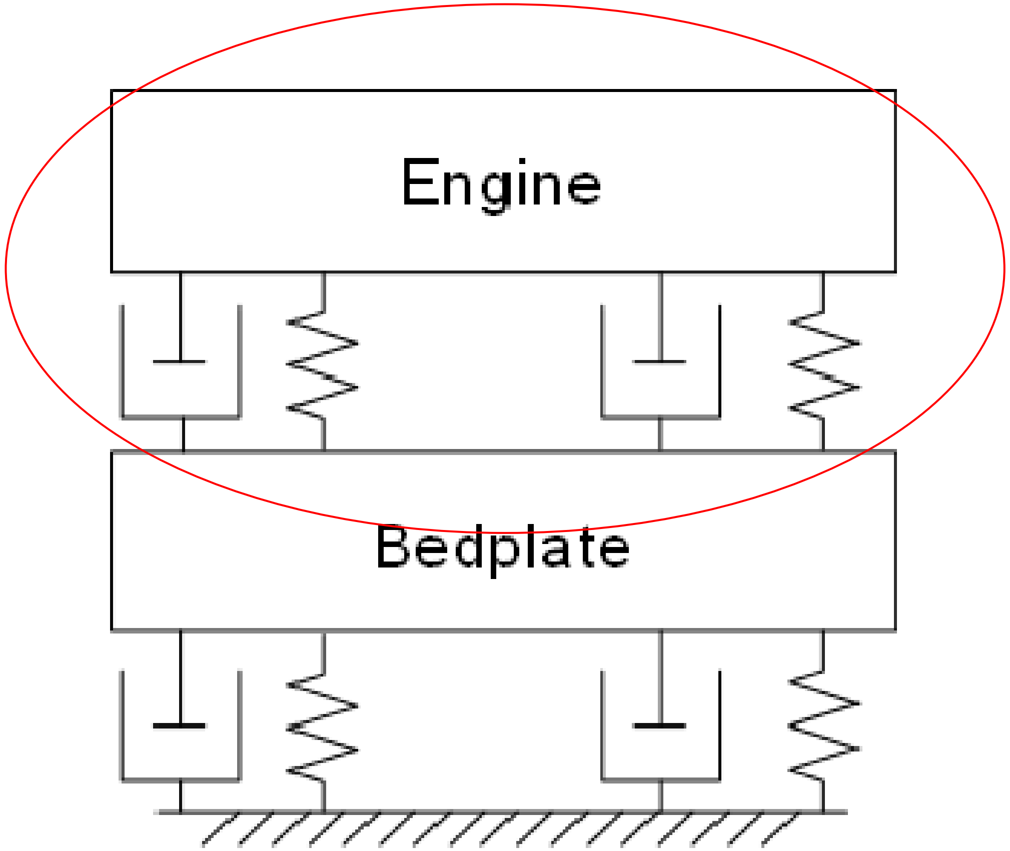
Mechanic Setup
Engine Mount

Trolley Mount



Mechanic Setup
Engine Mount

2 Mass Oscillator (deformation of bedplate neglected)



- First and Second Order Mass Forces

$$F_m = \underbrace{mr\omega^2 \cos(\omega t)}_{1st \quad Order} + \underbrace{mr\omega^2 \lambda \cos(2\omega t)}_{2nd \quad Order} + \ldots$$

m:	oscillating mass in the engine (piston and partially rod)
r:	crank radius
λ:	crank radius / rod length

- Gas Forces

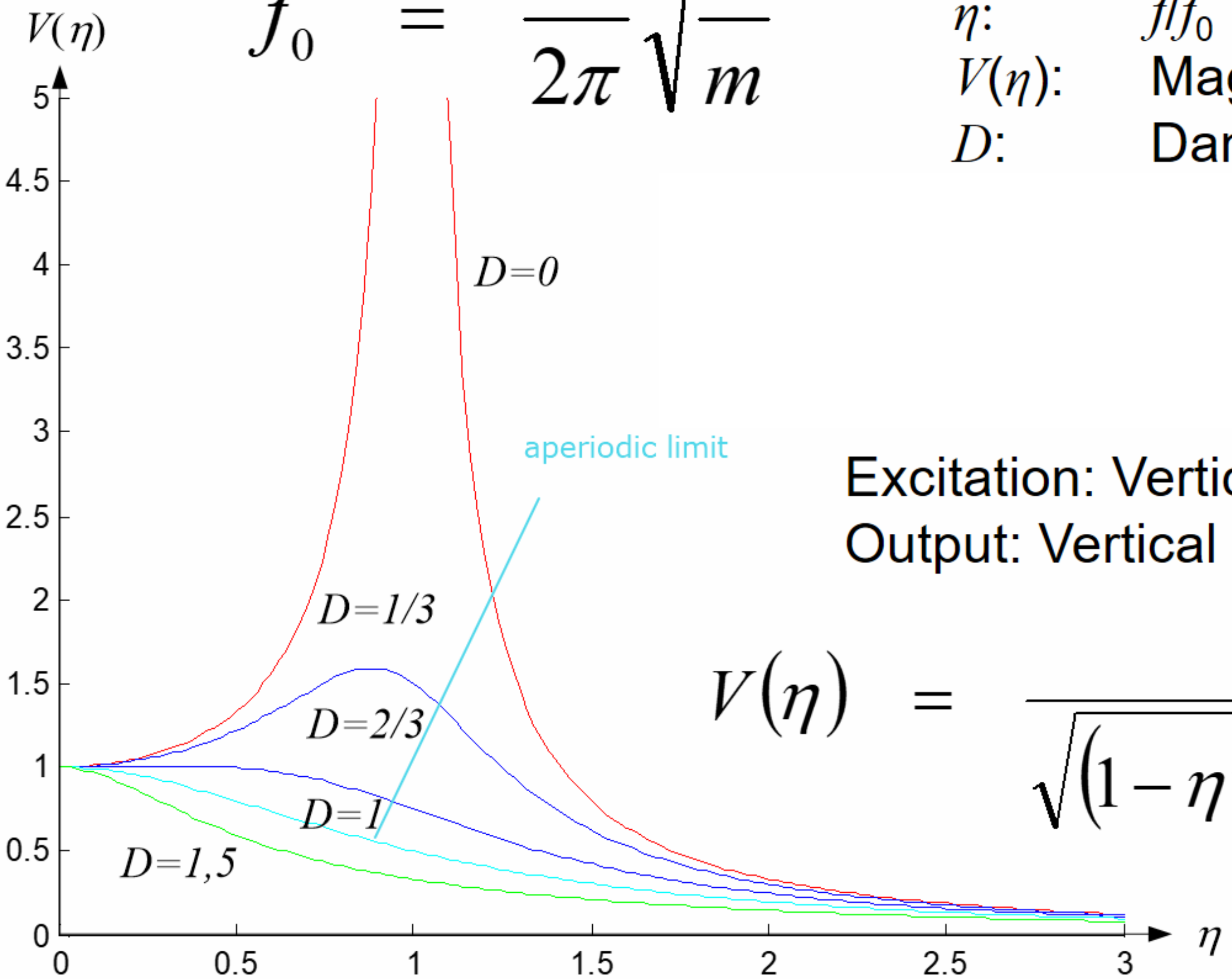
Resulting Forces and Torques

	F_1	F_2	M_1	M_2
3 Cylinders in Line	0	0	$\sqrt{3}F_1a$	$\sqrt{3}F_2a$
4 Cylinders in Line	0	$4F_2$	0	0
4 Cylinders Boxer	0	0	0	$2F_2a$
5 Cylinders in Line	0	0	$0.449F_1a$	$4.98F_2a$
6 Cylinders in Line	0	0	0	0
6 Cylinders V90°, 3 Throws	0	0	$\sqrt{3}F_1a$	$\sqrt{6}F_2a$

a: Distance between 2 Cylinders

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{c}{m}}$$

η: f/f_0
$V(\eta)$: Magnifier
D: Damping

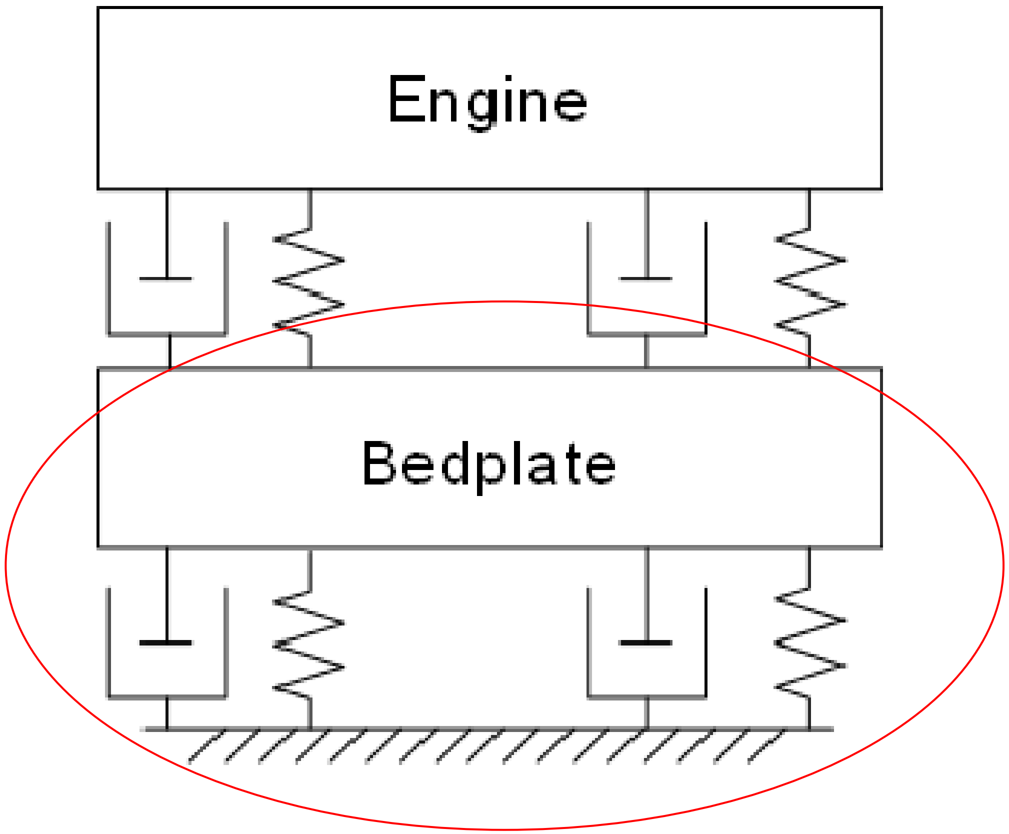


Excitation: Vertical Engine **Force**
Output: Vertical Engine **Dislocation**

$$V(\eta) = \frac{1}{\sqrt{\left(1-\eta^2\right)^2 + \left(2D\eta\right)^2}}$$

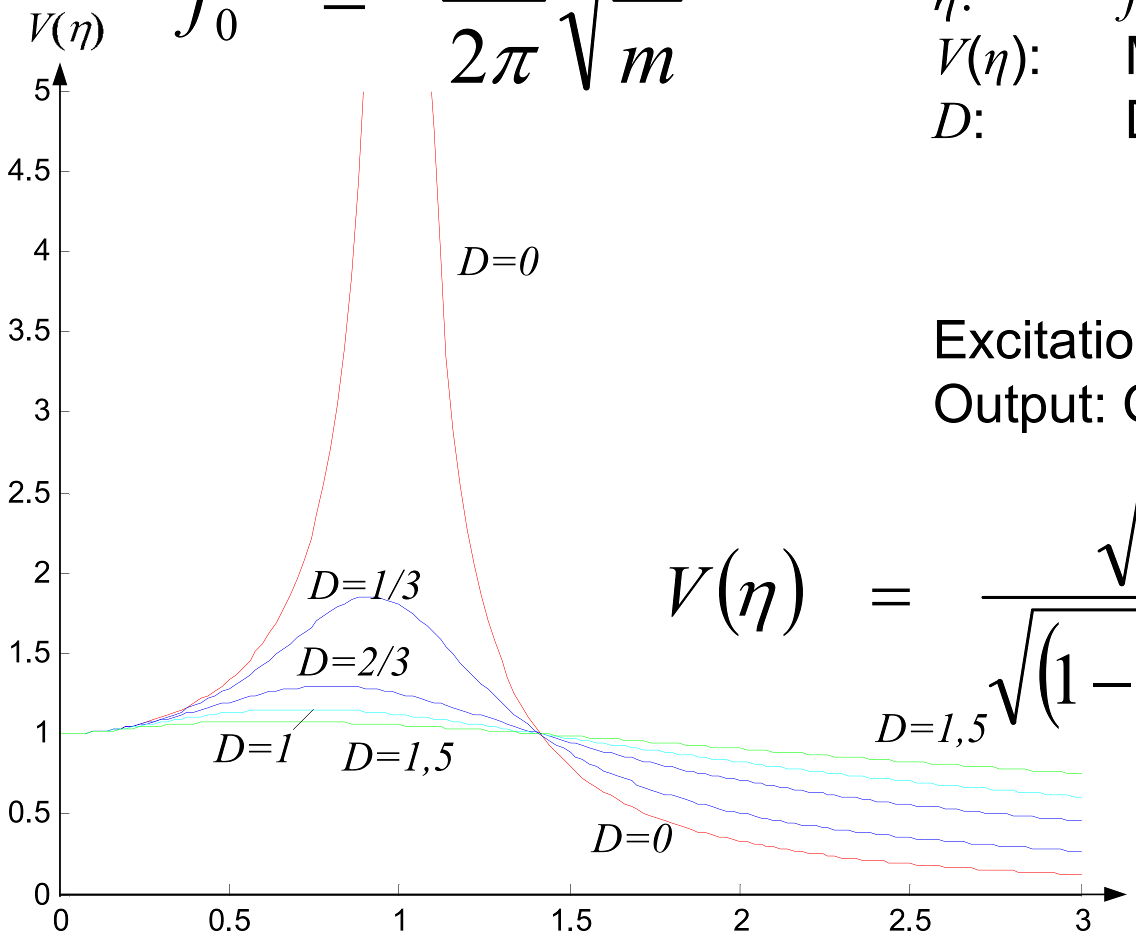
Mechanic Setup
Engine Mount

2 Mass Oscillator (deformation of bedplate neglected)



$$f_0 = \frac{1}{2\pi}\sqrt{\frac{c}{m}}$$

η: f/f_0
$V(\eta)$: Magnifier
D: Damping



Excitation: Plate **Force**
Output: Ground **Force**

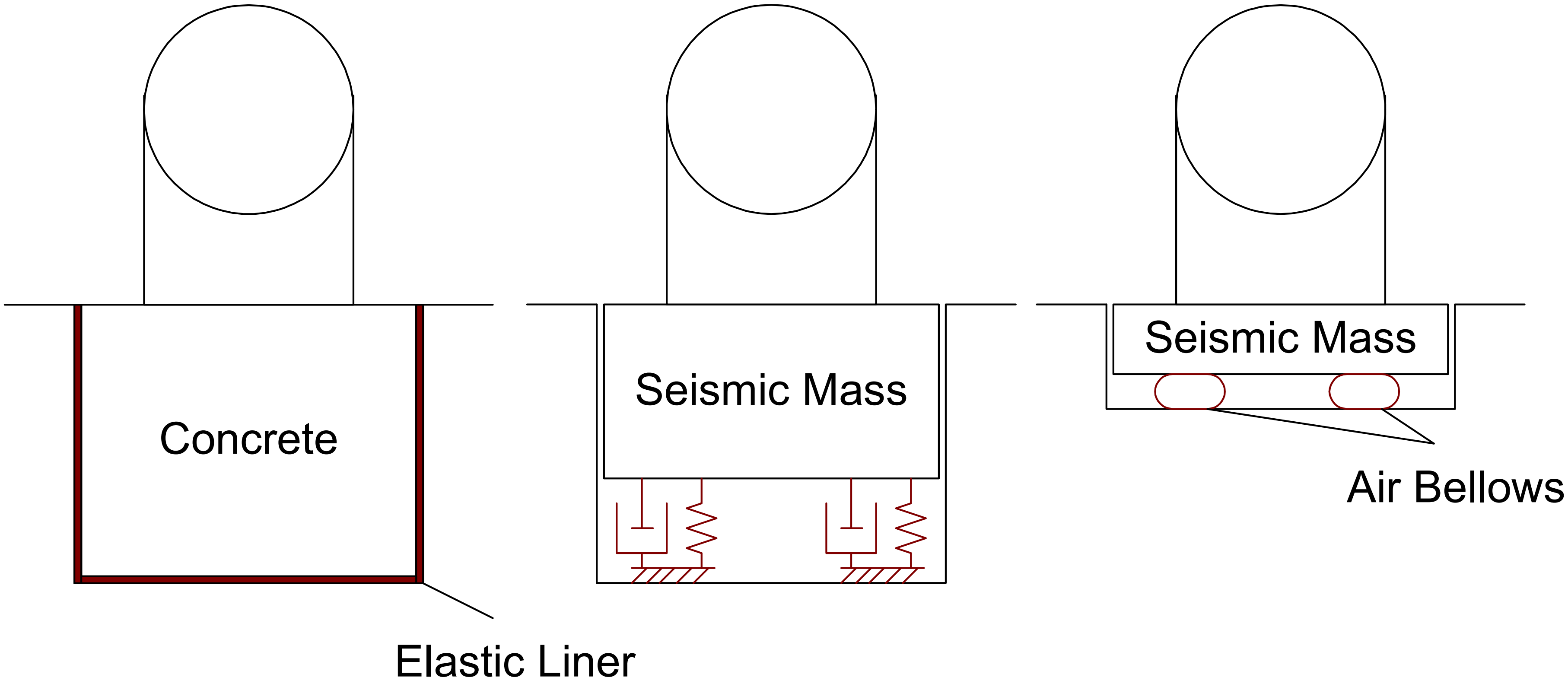
$$V(\eta) = \frac{\sqrt{1+(2D\eta)^2}}{\sqrt{(1-\eta^2)^2+(2D\eta)^2}}$$

Mechanic Setup
Elastic Bedplate Mounting

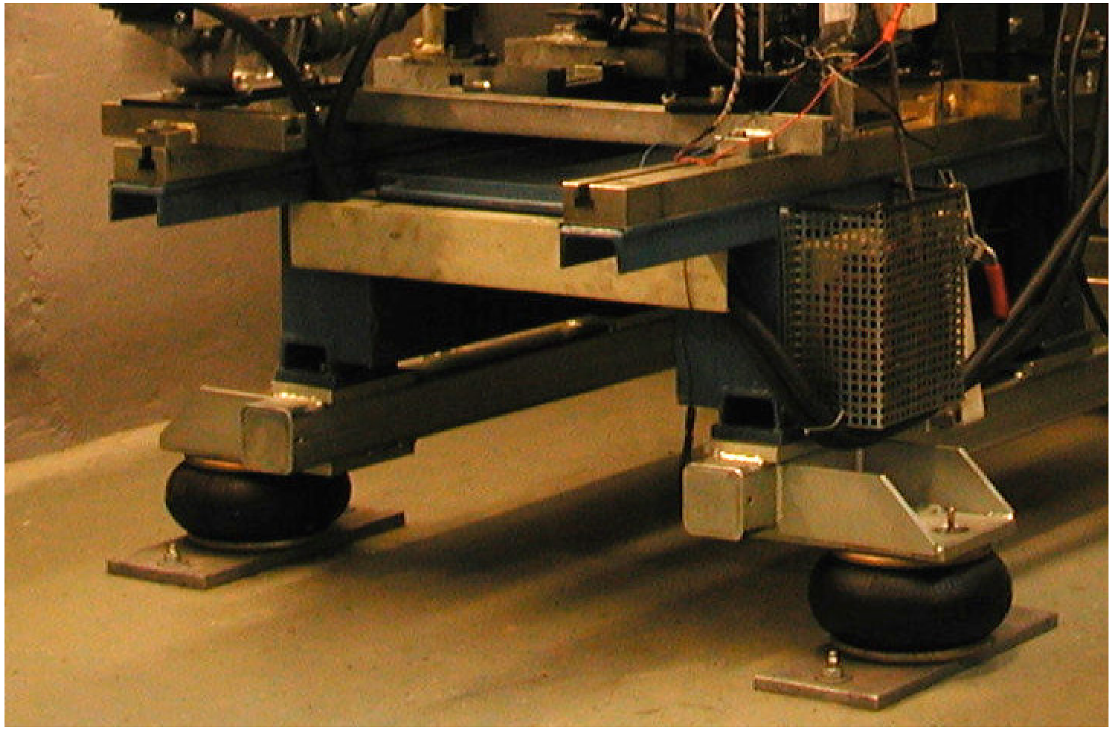
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{c}{m}}$$

C: Stiffness of Spring/Elastic Material

m: Seismic Mass



Mechanic Setup
Air Bellows

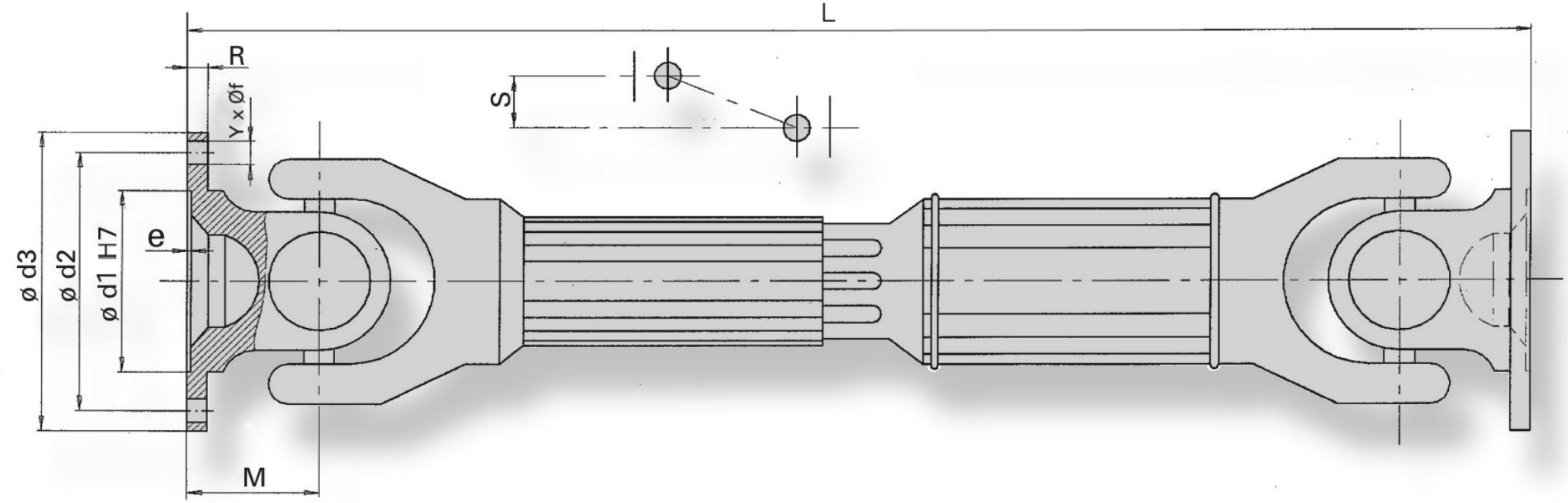


Source:
Elring Klinger, Idstein

4.1 Engine and Dyno Mount, Longitudinal Oscillations

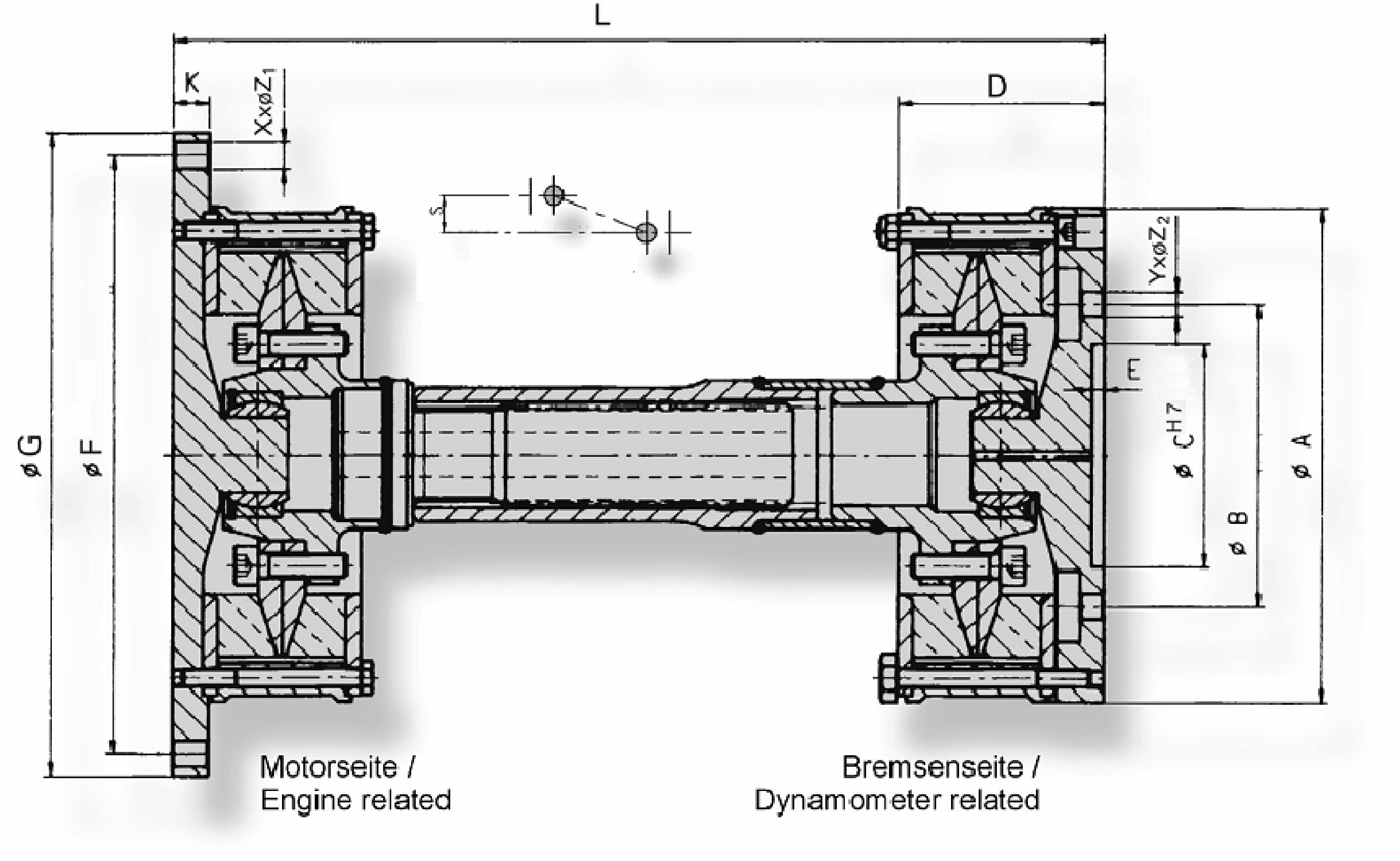
4.2 Shaft, Torsional Oscillations (and Bending Oscillations)

Mechanic Setup
Joint Shaft



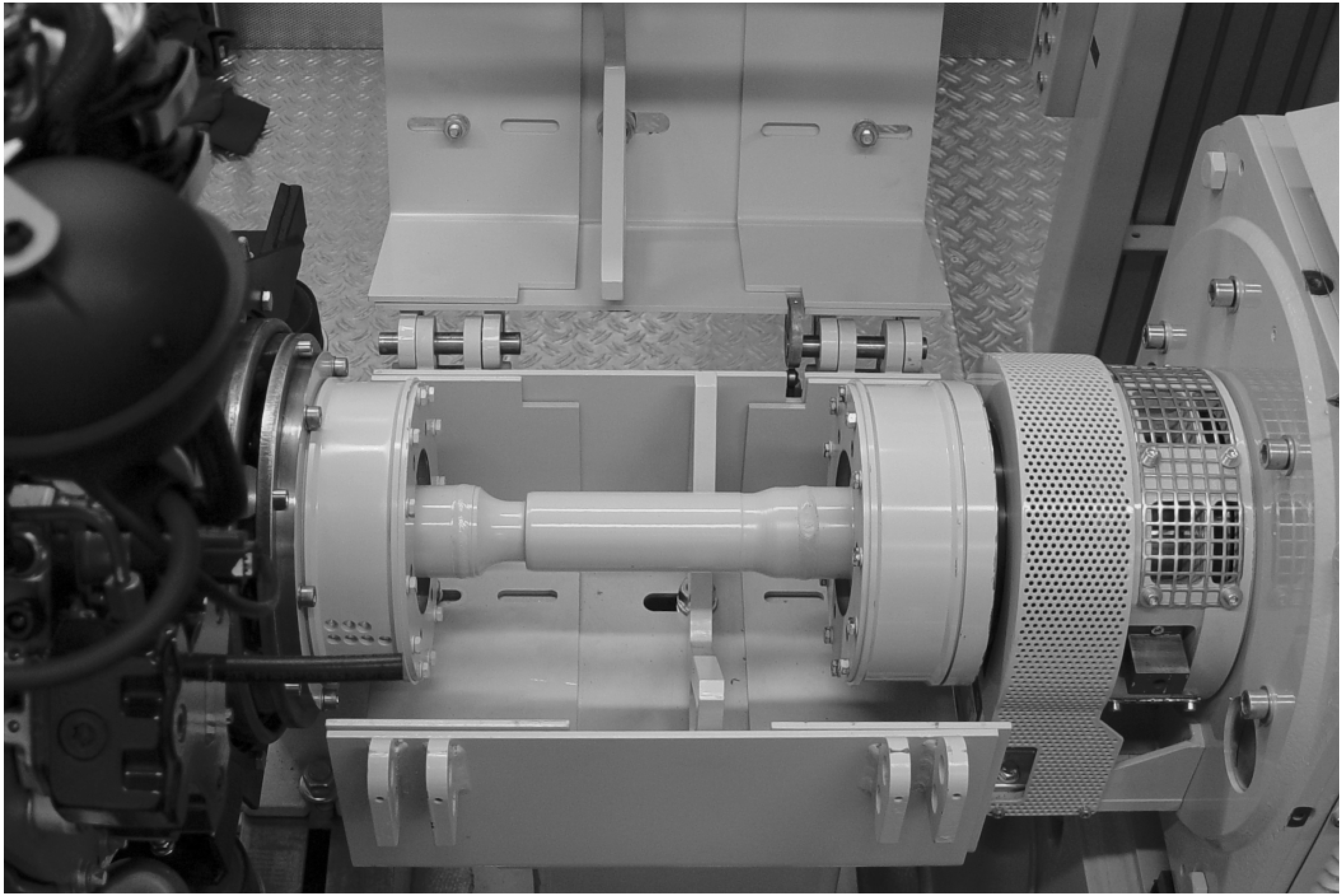
Source: Horiba

Mechanic Setup
Elastic Shaft

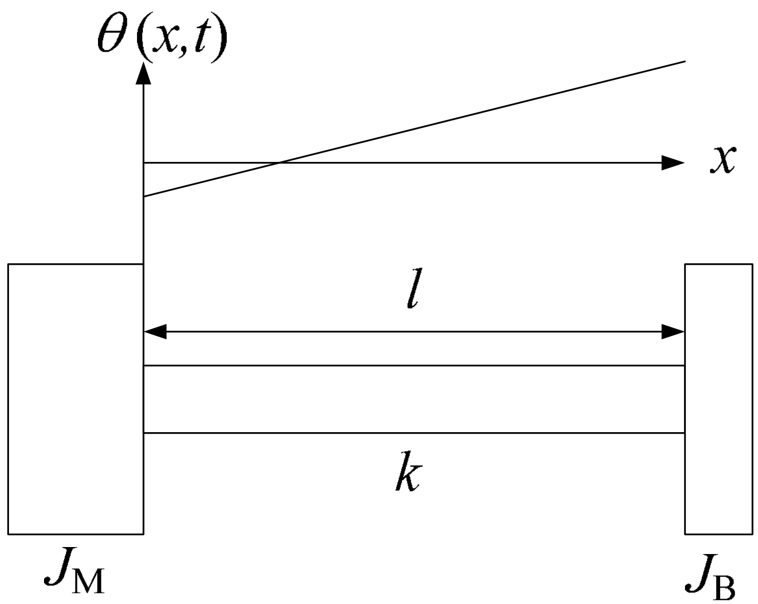


Source: Horiba

Mechanic Setup
Elastic Shaft with Shaft Protection



Simple Calculation Model

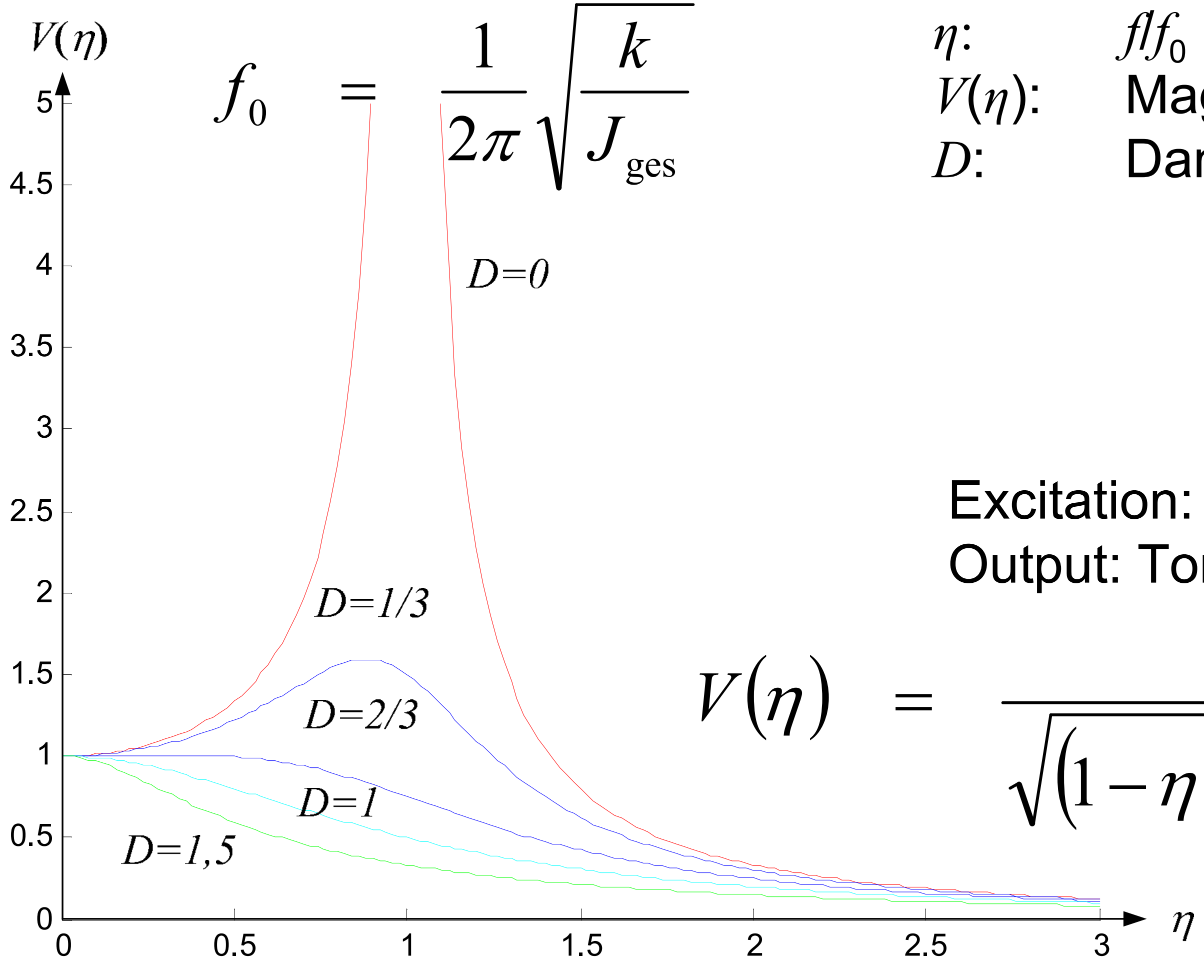


$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{J_{ges}}}$$

$$J_{ges} = \frac{J_M J_B}{J_M + J_B}$$

Mechanic Setup
Shaft

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{J_{ges}}}$$



η: f/f_0

$V(\eta)$: Magnifier

D: Damping

e.g. $J_1 = 1\,kgm^2$, $J_2 = 0{,}32\,kgm^2$

$k = \frac{1700\,Nm}{rad}$

$\Rightarrow f_0 = 13{,}7\,Hz$! Danger!

Excitation: Torque

Output: Torsion

$$V(\eta) = \frac{1}{\sqrt{\left(1-\eta^2\right)^2 + \left(2D\eta\right)^2}}$$

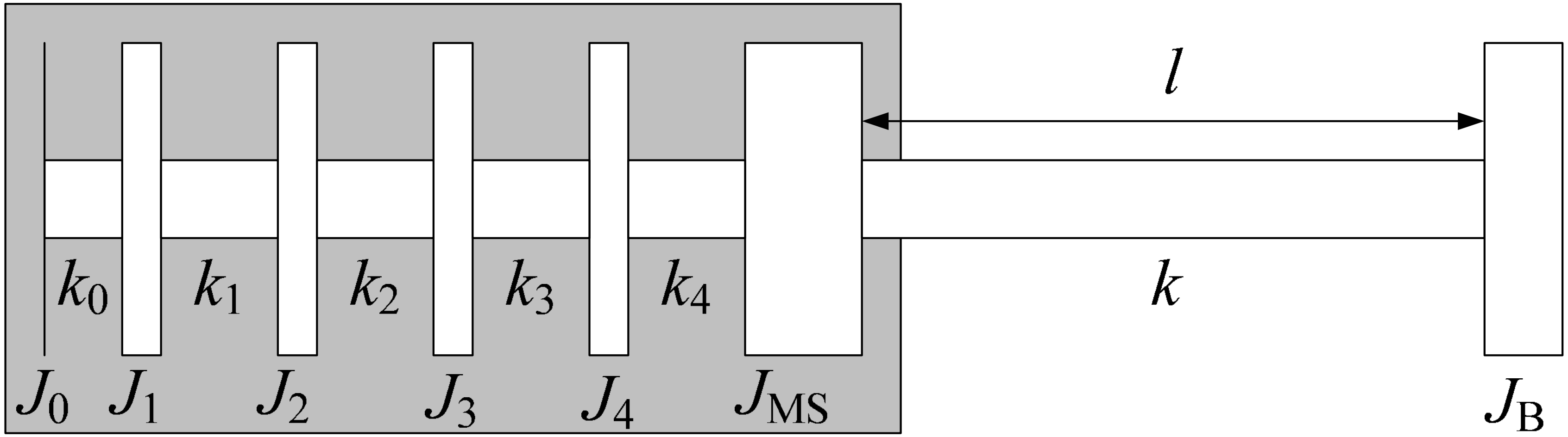
Mechanic Setup
Shaft

Why does the engine torque oscillate?

Why does the engine torque oscillate?

Mechanic Setup
Shaft

Detailed Calculation Model



$$\mathbf{K}\underline{\theta} + \mathbf{D}\underline{\dot{\theta}} + \mathbf{J}\underline{\ddot{\theta}} = \underline{M}$$

(numerical solution recommended)

$$\mathbf{K} = \begin{pmatrix} k_0 & -k_1 & 0 & 0 & 0 & 0 & 0 \\ -k_1 & k_0+k_1 & -k_2 & 0 & 0 & 0 & 0 \\ 0 & -k_2 & k_1+k_2 & -k_3 & 0 & 0 & 0 \\ 0 & 0 & -k_3 & k_2+k_3 & -k_4 & 0 & 0 \\ 0 & 0 & 0 & -k_4 & k_3+k_4 & -k_5 & 0 \\ 0 & 0 & 0 & 0 & -k_5 & k_4+k & -k \\ 0 & 0 & 0 & 0 & 0 & -k & k \end{pmatrix}$$

$$\mathbf{J} = \begin{pmatrix} J_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_{MS} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_B \end{pmatrix}$$

Shaft (Rotational Stiffness)

If no data sheet is available:

$$\frac{M}{\theta} = K = \frac{GI_{\mathrm{p}}}{l}$$

M:	torque	G:	shear modulus
θ:	torsion	I_{p}:	polar second moment of area
K :	rotational stiffness	l:	length

$$G = \frac{E}{2(1+\upsilon)}$$

E:	Young’s modulus	υ :	Poisson’s ratio

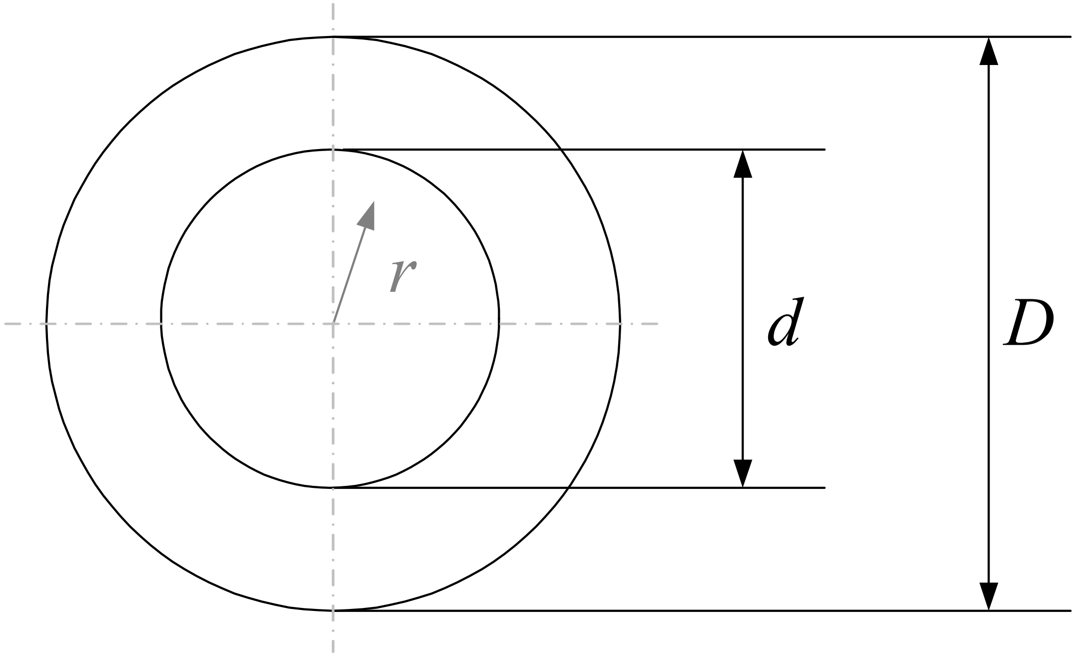
Examples: G_{steel} = 80 GPa, G_{Al} = 26 GPa

Polar Second Moment of Area

$$I_{\mathrm{p}} = \int r^2 dA$$

$$= \frac{\pi\left(D^4 - d^4\right)}{32}$$

(possibly d =0)



Inhomogeneous Shaft

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \ldots + \frac{1}{K_n}$$

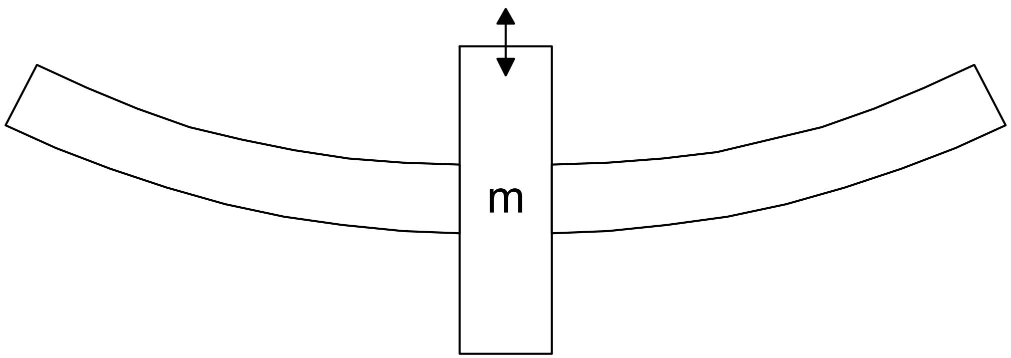
Example:

Additional Torque Measurement Flange

$$K_{\mathrm{shaft}} = \frac{1.7\,\mathrm{kNm}}{\mathrm{rad}}, \quad K_{\mathrm{HBM\,T10F500\,Nm}} = \frac{1000\,\mathrm{kNm}}{\mathrm{rad}}$$

Bending Oscillations
(in Practice Less Critical than Torsional Oscillations)

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{c}{m}}$$



f_0: resonant frequency
c: stiffness
m : mass

Mechanic Setup
Shaft

Sometimes additionally necessary:

- Clutch
- Gearbox
- Flywheel (normally the engine flywheel is used)