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Numerical Methods and Simulation
Module MT 03
Winter term 2021/22

https://moodle.th-ab.de/course/view.php?id=2907

Problem Set B

Seperable and exact ODEs

Homework

Exercise 1:

Solve the following ODEs:

(a) $y' \sin y + x = 0$

(b) $y' = -4x\sqrt{y-1}$ for $y \geq 1$

(c) $y'(1+x^2)x + 2y = 0$

Exercise 2:

Solve the following ODEs by substitution:

(a) $y' = 4x - y$

(b) $xy' = y(\ln(y) - \ln(x))$ for $x, y > 0$

(c) $\frac{dy}{dx} = -\frac{x+y}{3x+3y-4}$

Exercise 3:

Solve the following initial value problems:

(a) $y^2y' + x^2 = 1$ with $y(2) = 1$

(b) $x(x+1)y' = y$ with $y(1) = \frac{1}{2}$

Exercise 4:

Check if the following ODEs are exact or can be made exact by an integrating factor.
If possible solve the ODE.

(a) $(x^2 + 4xy)\ dx\ +\ (2x^2 - y^2)\ dy\ = 0$

(b) $y(2x^2y^2 + 1)y' + x(y^4 + 1) = 0$

(c) $2xy' + 3x + y = 0$

Exercise 5:

Solve the following inhomogeneous ODE for a parameter $a \neq 0$. What is the dimension of the solution space?

$$\frac{dy}{dx} + \frac{xy}{a^2 + x^2} = x$$

Exercise 6:

The Bernoulli equation is very similar to a linear ODE. It has the general form

$$\frac{dy}{dx} + p(x)y = q(x)y^n \qquad \text{with } n \neq 0, 1$$

Use the substitution $v = y^{1-n}$ to turn the nonlinear ODE in y into a linear ODE in v and solve the linear as well as the original ODE.