

Organizational Matters

Problem Set A

Calculation of Fourier coefficients

Homework

Exercise 1:

Consider the function $f(x) = x$ with $0 \leq x \leq 2\pi$ which is periodically continued on all \mathbb{R} .

- Sketch the function
- Represent the function by a Fourier series
- Discuss the convergence of the Fourier series at the point $x = 0$.
- Explain the values of the coefficients a_k .

Exercise 2:

Consider the 2π -periodic time signal

$$f(t) = \begin{cases} \cos(t) - \frac{1}{2} & 0 \leq |t| \leq \frac{\pi}{3} \\ 0 & \frac{\pi}{3} < |t| \leq \pi \end{cases}$$

- Sketch the signal
- Represent the function by a Fourier series
- Calculate the first Fourier coefficients up to $k = 2$ explicitly.
- Discuss the convergence of the Fourier series at the point $x = 0$.

Note: $\cos(\alpha)\cos(\beta) = \frac{1}{2}(\cos(\alpha + \beta) + \cos(\alpha - \beta))$

Exercise 3:

Consider the 2π -periodic function

$$f(x) = \begin{cases} \sin|x| & 0 \leq |x| \leq \frac{\pi}{2} \\ 1 & \frac{\pi}{2} < |x| \leq \pi \end{cases}$$

- Sketch the function
- Represent the function by a truncated Fourier series up to $k = 2$.

Note: $\cos(\alpha)\sin(\beta) = \frac{1}{2}(\sin(\alpha + \beta) - \sin(\alpha - \beta))$

Exercise 4:

Consider the 2π -periodic time signal

$$f(t) = \begin{cases} |t| & 0 \leq |t| \leq \frac{\pi}{2} \\ |t| + 1 & \frac{\pi}{2} < |t| \leq \pi \end{cases}$$

- Sketch the signal
- Represent the signal by its Fourier series and make the Fourier coefficients up to $k = 3$ explicit.
- Against which value converges the Fourier series at $x = \pi/2$?