Aschaffenburg University of
Applied Sciences
Faculty for Engineering
Prof. Dr. Michael Möckel

Numerical Methods
and Simulation
Module MT 03
Winter term 2015/16

Example questions for an exam

on ... January 2016

Personal details[1]:

Immatriculation number:												

First and last name:	

Remarks:

(1) The duration of the exam is 90 minutes.

(2) Be aware that you may find a later question easier than an earlier one.

(3) If not stated otherwise a subquestion is valid three points. Full score is only awarded if and only if all correct answers are given and no wrong answer is given.

(4) Please use a black or blue pen for your answers. Pencil will not be accepted.

(5) The use of any electronic device except a non-programmable calculator is not permitted. Please switch them off.

(6) No materials other than one sheet (two pages) of hand-written notes are permitted.

(7) You are free to answer all questions in English, German or any commonly understandable mix of both languages.

Results:

Question:	1	2	3	4	5	TOTAL
Achievable:						
Achieved:						

Mark:

[1] Your name is optional

Question 1: Basic Matlab

Q	Max	Achieved
1	14	

(a) In Matlab code `a=rand(20,1)` and `A=rand(20,20)` are a vector a and matrix A of random numbers. What are the dimensions of the results r for the following expressions?

Expression:	diag(a)	A(7:12, 9:20)	A * a
Dimensions:	(20,20)	(6,12)	(20,1)

(b) Define a vector `x` in Matlab code which represents all even integers for 2 to 2000.

```
x = 2:2:2000
```

(c) Implement the following function as an anonymous function in Matlab code such that it can be called both for single numbers and vectors:

$$f(x) = (\tan(x) + 1)\ln(x^2 + 5)$$

```
f = @(x) (tan(x)+1).*log(x.^2+5)
```

(d) Define the above function as an inline function.

```
f = inline('(tan(x)+1).*log(x.^2+5)','x')
```

(e) Write a Matlab script that calculates the mean value of $N \in \mathbb{N}$ random numbers. Do not use loops.

```
x = rand(N,1);
mean(x);
```

(f) (6 points) Write a Matlab script which plots the curve of descending parachute from $t = 0$ until it reaches z=0. $x(t) = 2\cos(t), \quad y(t) = 2\sin(t), \quad z(t) = 10 - t$

```
t = 0:0.1:10;
plot3(2*cos(t), 2*sin(t), 10-t);
```

Q	Max	Achieved
2	14	

Question 2: Root finding

Bisection is a standard approach to find a root of a function. Consider the function $f(x) = e^{-x^2} - 1/2$. It has a root around x=0.83.

(a) Under which requirements does the bisection algorithm work?

If root is single root, i.e. $f(x_r + \delta) \cdot f(x_r - \delta) < 0$

Does this requirement hold for the current example (Y or N)? Y

(b) Describe the update mechanism of the bisection algorithm.

Start with interval. Calculate midpoint. Evaluate $f(x)$ at midpoint. Update interval by replacing x_L or x_R s.th. $[x_L, x_R]$ contains single root

(c) Run the algorithm for four iterations by hand and fill in the results in the following tables (precision 2 digits).

Iteration i	x_L	x_R	$f(x_L)$	$f(x_R)$	m	$f(m)$
1	0.0	1.0	1/2	-0.13	0.5	0.27
2	0.5	1.0	0.27	-0.13	0.75	0.0698
3	0.75	1.0	0.0698	-0.13	0.875	-0.035
4	0.75	0.875	0.0698	-0.035		

What is the final error?

error:
$\Delta\varepsilon \approx 0.875 - 0.75 = 12.5\,\%$

Question 3: Ordinary differential eqns

Q	Max	Achieved
3	6	

Classify the following differential equations. Enter their order into the following table and tick the relevant box if an equation is linear, explicit or autonomous. If you want to correct a result use the last line of the table.

No	Ordinary differential equation	order	linear	explicit	autonomous
1	$\frac{dy}{dx} = ay$	1	Y	Y	Y
2	$\frac{d^2y}{dx^2} + (\frac{dy}{dx} - y)^2 = 0$	2	N	Y	Y
3	$\frac{d^2y}{dx^2} + (\frac{dy}{dx} - x)^2 = 0$	2	N	Y	N
	for corrections				

wg z^2 mit $z = \frac{dy}{dx}$