Aschaffenburg University of Applied Sciences
Faculty for Engineering
Prof. Dr. Michael Möckel

Email: Michael.Moeckel@th-ab.de
Telephone: +49 (0)6021 / 4206 - 507

Numerical Methods and Simulation
Module MT 03
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Problem Set 11

Introduction to Simulink

Exercise 1: Getting familiar with Simulink

Simulink is a Matlab tool for simulating dynamical systems and to investigate their behaviour. Simulation is w.r.t. an artificial 'time' coordinate which can be pseudo-continuous or explicitely discrete. It is useful for solving complex differential equations (continuous case) or finite difference equations (discrete case) numerically. Simulink comes with a user interface which allows for a special form of graphical programming of mathematical equations and is interfaced with Matlab. The graphical programming mimics an imaginative 'signal flow' from sources via operations to sinks.

(0) *Getting started.* Start simulink using the command `simulink` in the Matlab command window. Simulink is organized as a library of components and the Simulink Library Browser opens. Create a new model (green plus sign) in the toolbar. This opens the Simulink editor and a plain white worksheet. Use drag and drop to move items from the Library Browser to the worksheet.

(1) *Basic items.* The left window of the Simulink Library Browser shows an object tree. Pick from the menu 'Simulink' the submenu 'Sources' and choose 'Sin wave'. Click on the box and set as an initial condition 0. Add from the submenu 'Sinks' an object 'Scope' which allows to investigate functions w.r.t. simulation 'time'. Connect both by clicking – always opposite to the signal flow – first on the 'Scope' then on the 'Sin wave'. Start the simulation and click on the 'Scope'.

(2) *Integrator.* Pick 'Integrator' from the 'Commonly Used Blocks' submenu. It allows to integrate a signal over 'time'. At the input line of the integrator the order of the differential 'equation' is higher by one than at the output of the integrator. Connect the integrator to the source. To connect it to the 'Scope' the original signal and the output of the integrator need to be combined first. This is done using a 'Multiplexer (Mux)' block from the 'Commonly Used Blocks' submenu. Remove the line entering the 'Scope' and connect the source with the first input of the Mux. Then connect the integrator to the second output of the Mux. Connect the output of the Mux to the scope.

(3) *Initial conditions.* Implement the differential equation $dy/dx = \sin(x)$ with the initial condition $y(0) = -1$. This requires to specify the initial condition in the 'Integrator'. Click on its box and set its value. Global parameters of the simulation can be chosen in `Simulation - Configuration Parameters`. Set the integration range (start and stop time) to $[0, 4\pi]$. Run the simulation.

(4) *Labelling.* By clicking on the worksheet you can insert text labels. Use them a lot to make clear what the simulation does!

(5) *Higher order differential equations.* Solve the second order ODE $d^2y(x)/dx^2 + y(x) = 0$ with the initial conditions $y(0) = 1, dy/dx(0) = 0$. What is the analytical solution? Implement the equation in simulink as a closed loop diagram. Note: You need two integrators and an incremental feedback represented by 'Gain' from the 'Commonly Used Blocks' submenu. The value of the 'Gain' represents the prefactor of the signal (here: y) in the differential equation (here: 1). Connect to the 'Scope' at a point where the fully integrated signal is available.

(6) *Solution of the Logistic Equation.* The logistic equation is a differential equation which describes the growth of a population. γ is the growth rate proportional to the linear term, τ the loss rate proportional to the quadratic term.

$$\frac{dP(t)}{dt} - \gamma P(t) + \tau P^2(t) = 0$$

Classify what kind of differential equation is the logistic equation. Check that the analytical solution is

$$P(t) = \frac{P_0\gamma}{\tau P_0(1 - e^{-\gamma t}) + \gamma e^{-\gamma t}}.$$

Implement the logistic equation in simulink for the initial value $P(0) = 10000$ and the parameters $\gamma = 0.05$ and $\tau = 2.5e - 6$.

[1] O. Beucher, M. Weeks, Introduction to Matlab and Simulink, 3rd Edition, Infinity Science Press 2008

Exercise 2: Examples for ODEs in Simulink

Implement the following ODEs by drawing suitable diagrams in Simulink:

(1) Start with the third order ODE

$$\frac{d^3y}{dt^3} + a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = 0$$

(2) Implement the following nonlinear ODE in Simulink. Use the Math symbols product and square.

$$\frac{d^2y}{dt^2} = \left(\frac{dy}{dt}\right)^2 - ty$$

(3) Implement the following system of coupled ODEs in Simulink (i) directly by drawing (coupled) Simulink diagrams for each equation and then (ii) re-writing the system as a matrix ODE and drawing a single Simulink diagram for the matrix ODE.

$$\begin{aligned} \frac{d^2x}{dt^2} &= p\frac{dx}{dt} + ax + by \\ \frac{d^2y}{dt^2} &= q\frac{dy}{dt} + cx + dy \end{aligned}$$

(4) Simulink and Matlab can be coupled by calling Matlab functions in Simulink. Change the Simulink model in (1) to account for an external function $f(t)$ such that the ODE reads

$$\frac{d^3y}{dt^3} + a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = f(t)$$

The function $f(t)$ shall be defined in a MATLAB function file `f.m`. Include this function in Simulink by moving the *Matlab Function* symbol from the *User-Defined Functions* library onto the Simulink worksheet. Then click on the symbol such that the Matlab editor opens. Write a Matlab function which simply calculates $f(t) = \sin(t)$. Compare with the *Sin Wave* symbol from the *Sources* library.

Exercise 3: Van der Pol oscillator in MATLAB and Simulink

The Van der Pol oscillator is a simple model for nonlinear deviations from harmonic oscillations. It is given by the ODE (with $\mu \in \mathbb{R}$)

$$\frac{d^2x}{dt^2} + \mu(x^2 - 1)\frac{dx}{dt} + x = 0$$

(1) Discuss the difference of this ODE w.r.t. the (damped) harmonical oscillator.

(2) Implement the ODE in MATLAB. Re-write the ODE as a system of two coupled linear ODEs and define an anonymous function of the independent variable, the dependent (vector) variable and the parameter μ containing the right hand side of the system.

(3) Solve the ODE with a standard MATLAB solver (`ode45`) and plot the result as a phase portrait (x_1 vs x_2) for $\mu = 1$ and $\mu = 2$. Use the initial value $(x_1, x_2) = (1, 1)$ and optain a result for $t \in [0, 50]$.

(4) Now study the same problem for $\mu = 1000$. What do you observe? How would you proceed?

(5) Now implement the same ODE in Simulink and run the simulation. Present the results as a phase portrait (XY graph, time plot and an export to the MATLAB workspace.

(6) Write a control script in MATLAB which starts the simulation with differently depending on the given value of the parameter μ.

Exercise 4: Simulating a vibration damper with Simulink

Consider a mass M coupled to mechanical system of an ideal spring with spring constant k and a damper with damping constant b. An external force $F(t)$ acts on the system.

(1) Define a suitable second order ODE and two initial conditions $x_0 = 1$ and $v_0 = 0$. Use the parameters $M = 10$, $k = 1$, $b = 2$ and model the force as a sin wave with frequency $2\pi f = \omega = 1$ and amplitude F_0.

(2) Implement the system in Simulink and run it from a MATLAB control script. Study $F(t)$, $x(t)$and $v(t)$ for $F_0 = 0$ and $F_0 = 1$.